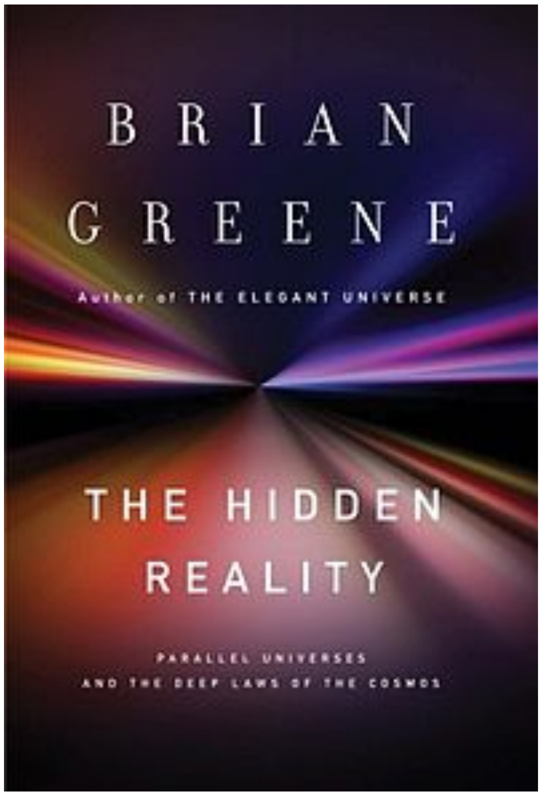
Brian Greene, Nov 6 @ 7pm

Elliott Hall of Music



Free seating pass required for entry

http://www.convocations.org/portfolio/brian-greene-11-6-14/#sthash.KqvAENA9.dpuf

Announcements

EXAM II is Tuesday

8-10 PM – TUESDAY, Nov. 4 in Elliot Hall of Music

Material: Chapters 17, 18, and 19 in the book.

How to Study:

Practice Exam + Solutions + Eqn Sheet is on BBL.

(Note the practice exam has some
Ch 20 material, but your exam will not)

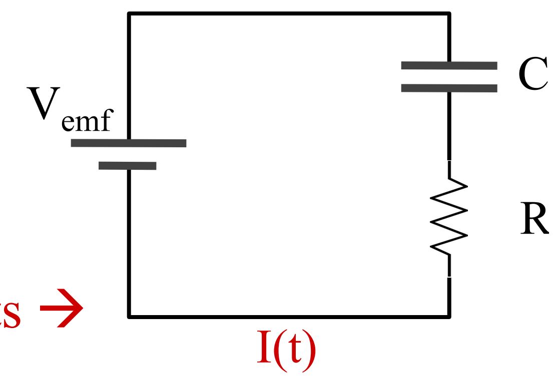
Find the WebAssign Extended Practice Problems
- not for credit, but you get 50 tries for ALL questions in Ch 17-19

Today

- Solving for Q(t) and I(t) in an RC circuit
- The "time constant" of an RC circuit is RC

GOAL: Find Q(t), I(t) in RC circuit

$$Q = CV$$



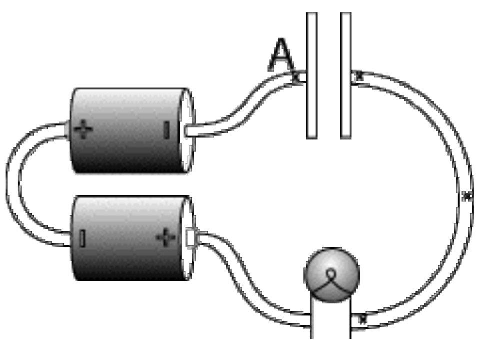
← Q is at the capacitor

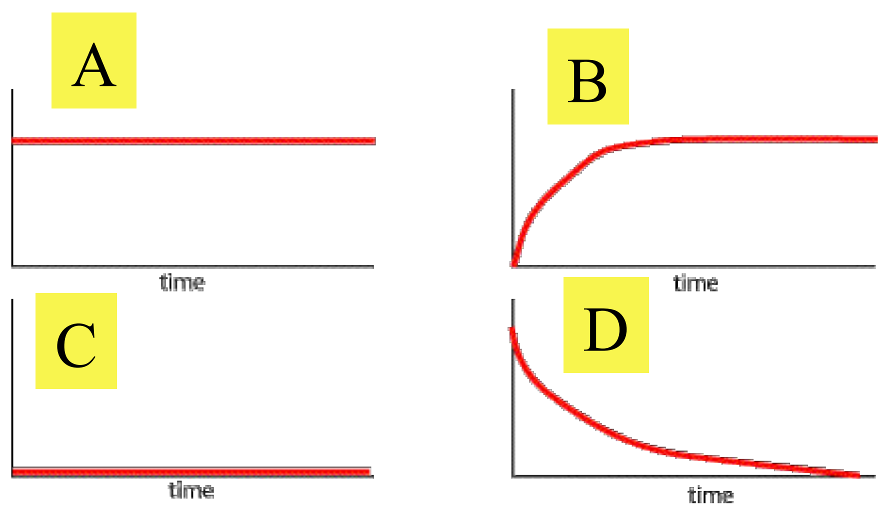
I is through all
circuit elements →

First: What do we expect?

Note: we use V_{emf}. Book calls it "emf".

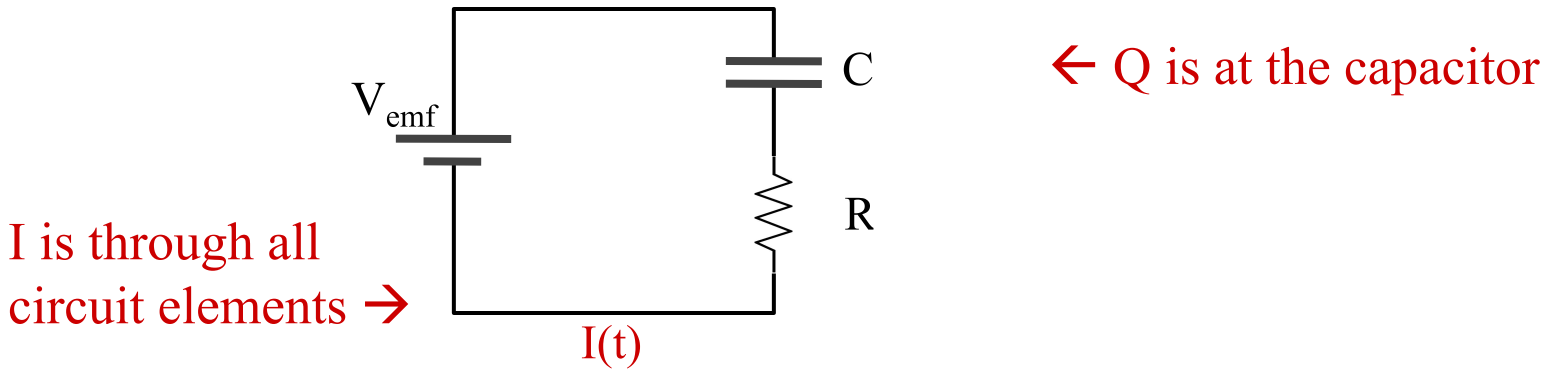
Capacitor initially uncharged. Which graph shows the magnitude of the
POTENTIAL DIFFERENCE across the LIGHT BULB FILAMENT while CHARGING?





GOAL: Find Q(t), I(t) in RC circuit

$$Q = CV$$



<u>First: What do we expect?</u>

Just after we connect the circuit:

$Q = 0$

$V_{emf} = I R$

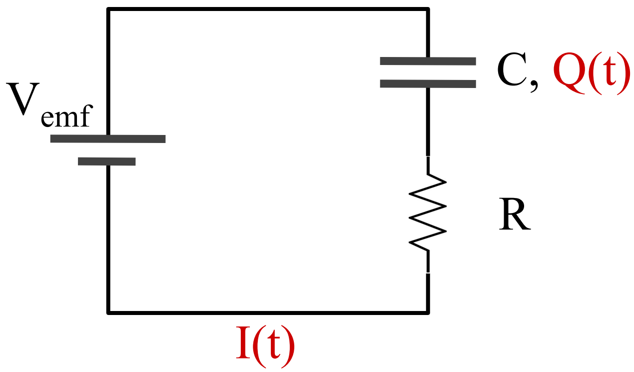
A long time after we connect it:

$Q = C V_{emf}$

$I = 0$

GOAL: Find Q(t), I(t) in RC circuit

$$Q = CV$$



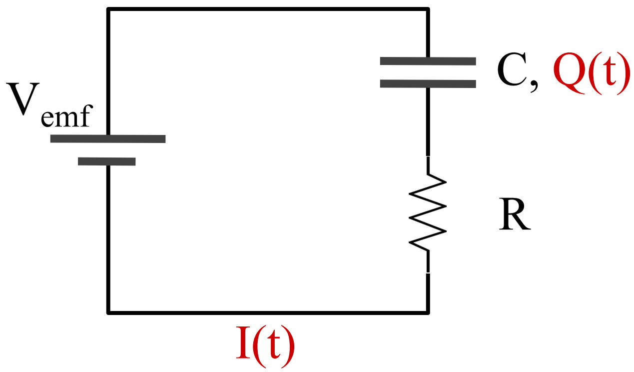
There's one more concept we need:

$$I = \frac{dQ}{dt}$$

Let's see how...

How are Q(t) and I(t) related?

$Q = CV$



In English: Current $I = |q|\ n\ A\ v$ is:
How much charge (ΔQ) passes by per unit time (Δt).

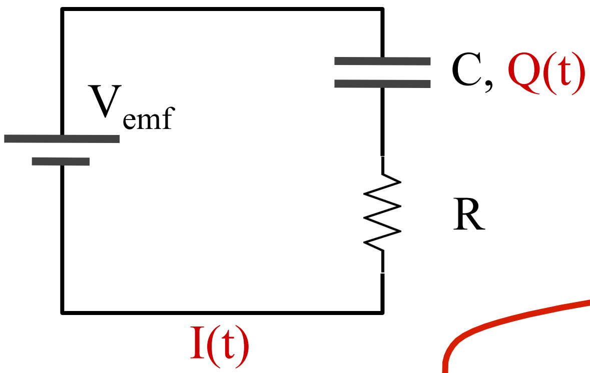
In Math:

$$I = \frac{\Delta Q}{\Delta t} \qquad lim\ \Delta t \to 0 \ \Rightarrow \boxed{I = \frac{dQ}{dt}}$$

Charge ΔQ per time Δt moves throughout the circuit,
but it **piles up** at C.
This is the same Q that is gathering on the capacitor.

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV \qquad I = \frac{dQ}{dt} \qquad V = IR$



$$V_{emf} = \frac{Q}{C} + IR$$

Apply Voltage Loop Rule

$$\boxed{V_{emf} = \frac{Q}{C} + \frac{dQ}{dt}R}$$

Use $I = \frac{dQ}{dt}$

Solve this Differential Equation for Q(t).

TIP: How do you "solve" a differential equation? By already knowing the answer!

We have:

$$\frac{dQ(t)}{dt} = \frac{V_{emf}}{R} - \frac{1}{RC}Q(t)$$

And the solution is:

$$\boxed{Q(t) = Ae^{-t/RC} + \text{constant}}$$

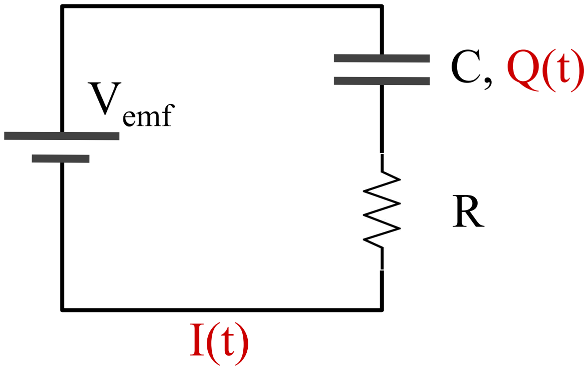
to be determined

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV$ $\quad I = \frac{dQ}{dt}$ $\quad V = IR$

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$



1. Try the "solution"

Use this

$$\frac{dQ}{dt} = -\frac{1}{RC}Ae^{-t/RC} = -\frac{1}{RC}\Big(Q(t) - \text{constant}\Big)$$

$$= \frac{1}{RC}\text{constant} - \frac{Q(t)}{RC}$$

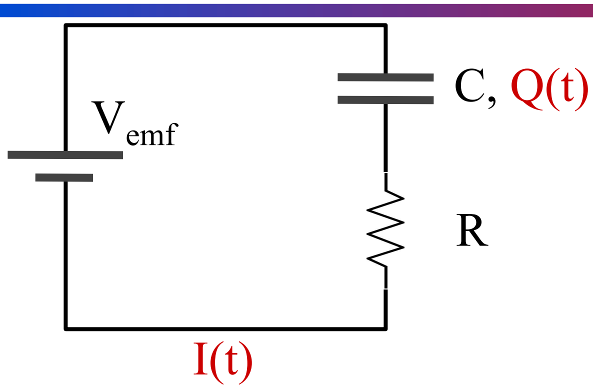
GOAL: Find Q(t), I(t) in RC circuit

Q = CV $\quad I = \frac{dQ}{dt} \quad$ V = IR

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$

Compare to this



1. Try the "solution"

$$\frac{dQ}{dt} = -\frac{1}{RC}Ae^{-t/RC} = -\frac{1}{RC}\Big(Q(t) - \text{constant}\Big)$$

$$= \frac{1}{RC}\text{constant} - \frac{Q(t)}{RC}$$

$$\Rightarrow \boxed{\text{constant} = V_{\text{emf}}C}$$

2. "Apply boundary conditions"
(*i.e.* use physics – think about the extremes)

At t=0: $\quad Q(t \to 0) = Ae^{o} + V_{\text{emf}}C = A + V_{\text{emf}}C = 0$

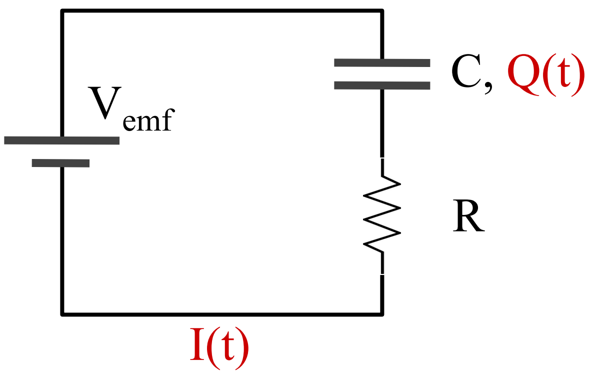
$$\Rightarrow \boxed{A = -V_{\text{emf}}C}$$

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV$ $\quad I = \frac{dQ}{dt}$ $\quad V = IR$

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$



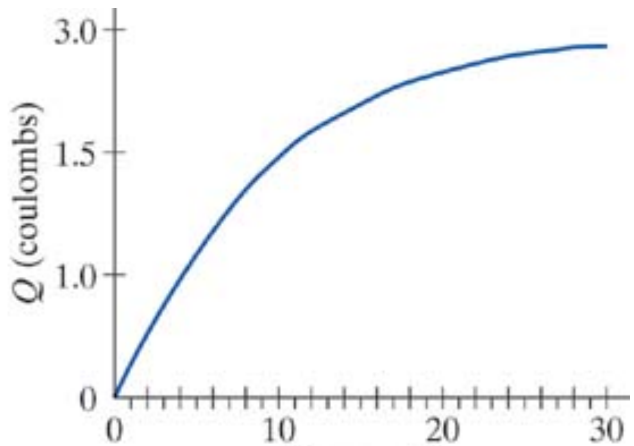
$$\text{constant} = V_{\text{emf}}C \qquad A = -V_{\text{emf}}C$$

$$Q(t) = -V_{\text{emf}}Ce^{-t/RC} + V_{\text{emf}}C$$

$$\boxed{Q(t) = V_{\text{emf}}C\left[1 - e^{-t/RC}\right]}$$

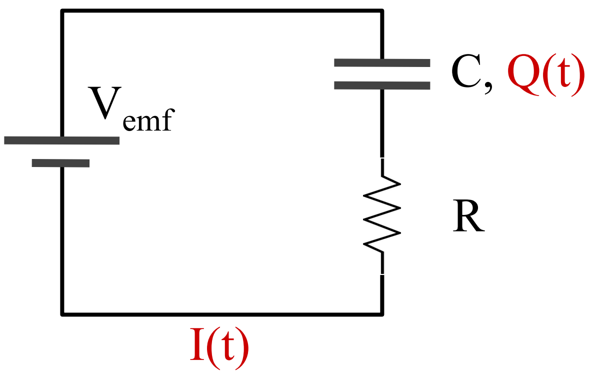
Doublecheck:
Is it what we expected?

✔ Yes, it is.



GOAL: Find Q(t), I(t) in RC circuit

$$Q = CV \qquad I = \frac{dQ}{dt} \qquad V = IR \qquad Q(t) = V_{\mathrm{emf}} C \left[1 - e^{-t/RC} \right]$$



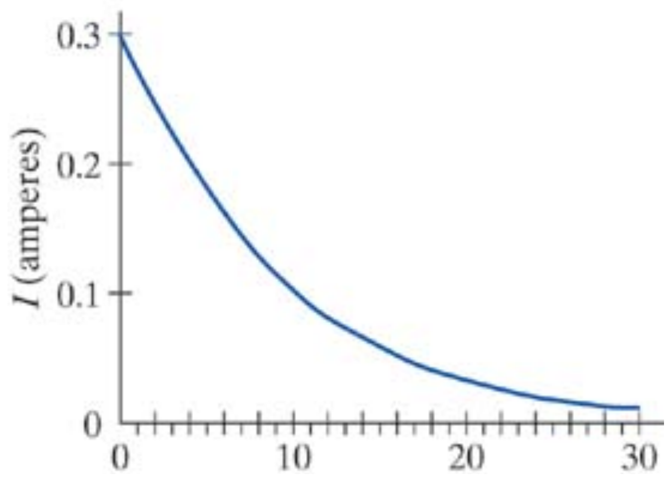
Now find I(t):

$$I(t) = \frac{dQ}{dt} = -V_{\mathrm{emf}} C \left[-\frac{1}{RC} e^{-t/RC} \right]$$

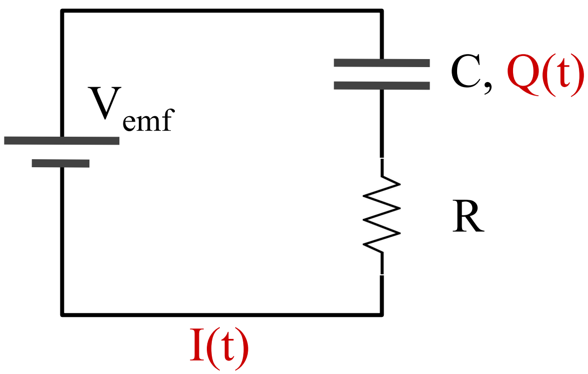
$$\boxed{I(t) = \frac{V_{\mathrm{emf}}}{R} e^{-t/RC}}$$

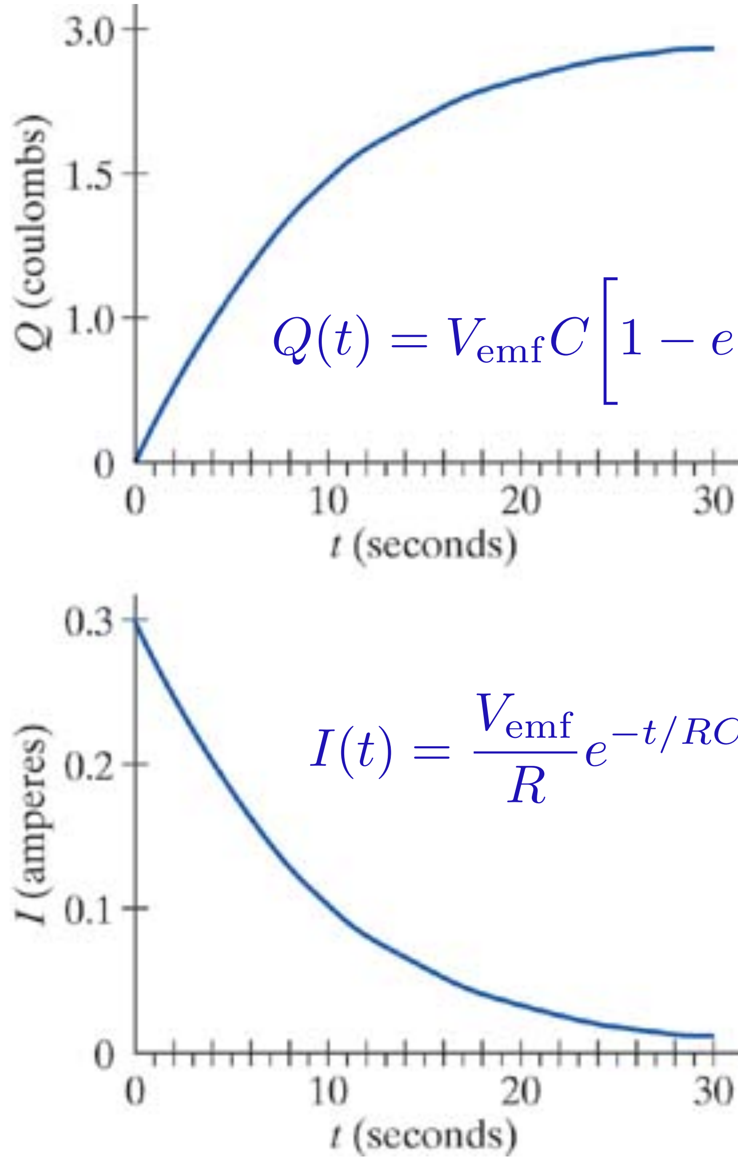
Doublecheck:
Is it what we expected?

✔ Yes, it is.



RC Circuit: Summary



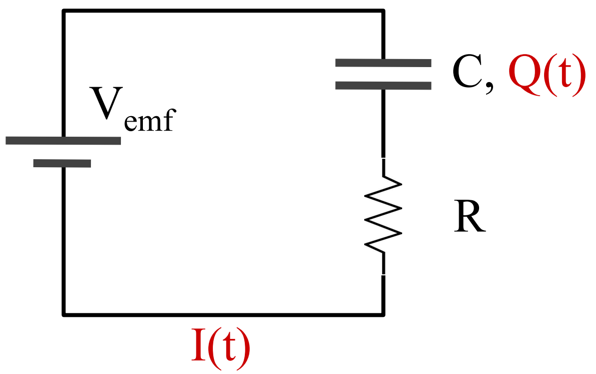


$$Q(t) = V_{\text{emf}} C \left[1 - e^{-t/RC} \right]$$

$$I(t) = \frac{V_{\text{emf}}}{R} e^{-t/RC}$$

The RC Time Constant

$$Q(t) = V_{\mathrm{emf}} C \left[1 - e^{-t/RC} \right] \qquad I(t) = \frac{V_{\mathrm{emf}}}{R} e^{-t/RC}$$



When time $t = RC$, the current I drops by a factor of e.

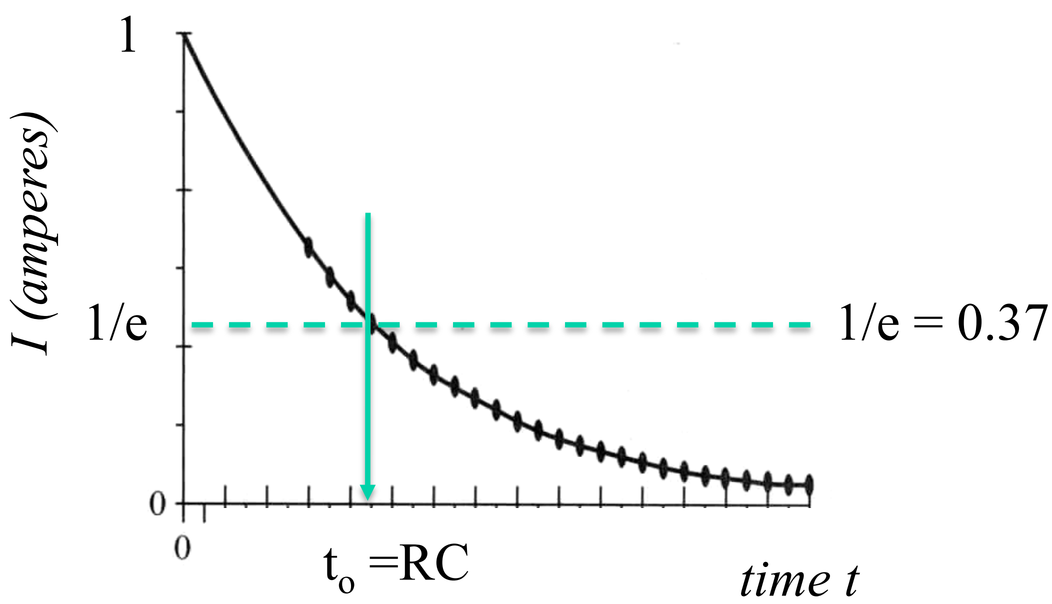
RC is the "time constant" of an RC circuit.

$$e^{-t/RC} = e^{-1} = \frac{1}{2.718} = 0.37 \quad \text{when } t = RC$$

A rough measurement of how long it takes to reach final equilibrium

What is the value of RC?

$$I(t) = \frac{V_{\rm emf}}{R} e^{-t/RC}$$

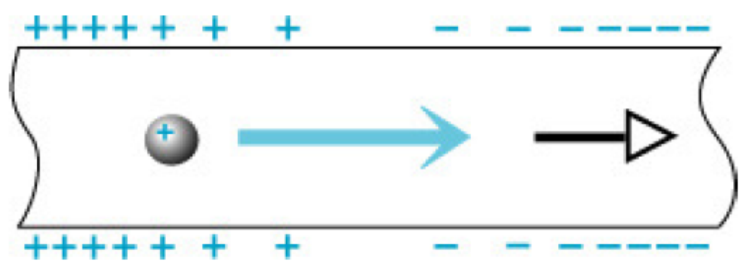


iClicker

This iClicker question is so hard that you will have to solve it by hand on the DocCam.

Current Density & Conductivity

$$\vec{J} = \sigma \vec{E}$$



carrier density

carrier charge

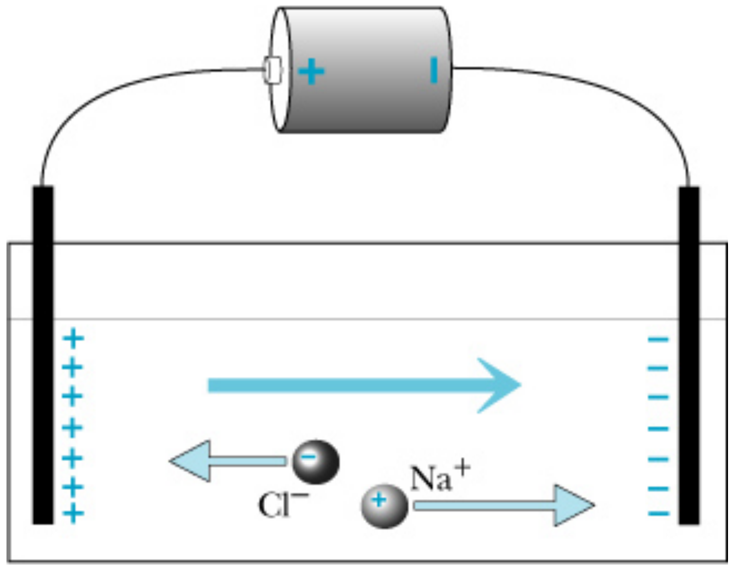
The conductivity is a material-dependent quantity: $$\sigma = |q|\, n\, u$$

mobility

The current density has magnitude $J = I / A$

Unit: J, A/m^2; E, V/m; σ, $(A/m^2)/(V/m)$

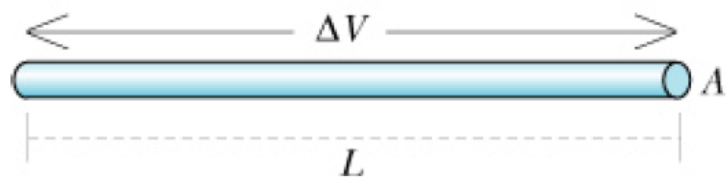
Conductivity with two kinds of charge carrier



$$\boxed{\sigma = |q_1| n_1 u_1 + |q_2| n_2 u_2}$$

Definition of Resistance

$$R = \frac{L}{\sigma A}$$



Important special case: if the conductivity remains constant regardless of how much current flows, then we call the material "ohmic" and any piece of the material obeys "Ohm's Law":

$$I = \text{constant} \;\; \text{x} \;\; |\Delta V|$$

Non-ohmic examples: diodes, capacitors, batteries.

"Ohmic" Resistors

$$|\Delta V| = I\ R$$

$$R = \frac{L}{|q|\,n\,A\,u}$$ *depends on both material properties and geometry*

Never write $V = I\ R$ *because …*

… what you really mean is $|\Delta V| = I\ R$

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